An Efficient Joint Timing and Frequency Offset Estimation for OFDM Systems

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***Abstract* -- This paper presents a new joint frame synchronization and frequency offset estimation algorithm for orthogonal frequency division multiplexing (OFDM) systems as a modification to Zhang's method [7]. By designing a new training preamble weighted by PN sequence, the timing estimator is improved (at least 3dB with low SNR). By estimating time offset first, the computational load is greatly reduced with no loss in frequency offset estimation accuracy. The performance of the proposed method is evaluated by computer simulations in terms of timing error rate (TER) together with computational complexity.**

***Index Terms* -- OFDM, timing synchronization, frequency offset estimation, computational complexity**

I. INTRODUCTION

ORTHOGONAL frequency division multiplexing (OFDM) has been widely used in communication systems such as digital audio broadcasting (DAB), digital video broadcasting (DVB), and asymmetric digital subscriber line (ADSL) modems together with wireless local area network (LAN) because of its robustness to inter symbol interference (ISI) and high efficiency in making use of bandwidth resources. However, it is very sensitive to synchronization errors caused by multipath delay and frequency shift as well as oscillator instability. To overcome this disadvantage, several approaches, data-aided or non-data-aided, have been proposed to perform timing synchronization and frequency offset estimation either jointly or individually.

Data aided schemes are more suitable for applications requiring fast and reliable synchronization. A popular algorithm to perform timing synchronization and frequency offset estimation jointly is proposed by Schmidl [1]. This algorithm has a timing metric, which is used to judge the beginning of the training symbol, robust to frequency offset. However, it has a plateau which introduces uncertainty when judging the starting point. Besides, two training symbols are required to estimate frequency offset, decreasing the overall system efficiency.

Kim's algorithm [2] uses only one symbol for both timing and frequency offset estimation. However, its frequency offset estimation is based on perfect symbol timing, an assumption which cannot always be guaranteed due to the plateau.

To eliminate the plateau inherent in Schmidl and Kim algorithm, several modifications are made. Minn's [3] method is free from the plateau. Nevertheless, the performance deteriorates when the number of subcarrier is small and/or in multipath fading channel. An approach proposed by Park [4] yields an impulse shaped timing metric, but it suffers from side-peaks. [5] and [6] propose timing and frequency offset synchronization methods for preambles with constant envelope.

Zhang's method [7] is able to get more accurate frequency offset estimation than that of Schmidl. This improvement, however, is at the expense of far heavier computational load. Furthermore, its timing estimator is not precise enough to ensure satisfactory operation of frequency offset estimator.

In this paper, a timing estimator with the training preamble weighted by PN sequence similar to [5] is adopted as a modification to Zhang's method. On one hand, it achieves better timing synchronization performance. On another hand, Zhang's algorithm is greatly simplified with no loss in frequency offset estimation accuracy.

This paper is organized as follows. Section 2 gives the system model. In Section 3, Zhang's algorithm is briefly described. Section 4 is about the proposed algorithm performing joint timing synchronization and frequency offset estimation. Section 5 gives simulation results and computational complexity analysis. Finally Section 6 concludes the whole paper.

II. SYSTEM MODEL

The OFDM samples at the outputs of the IFFT are given by

$$x(i) = \frac{1}{\sqrt{N}}\sum_{k=0}^{N_u-1} X_k e^{j2\pi ik/N},\ i = 0,1,2 \dots, N-1 \qquad (1)$$

where X_k is the complex modulated symbol on the k-th subcarrier, N_u and N is the size of subcarriers and IFFT, respectively. An OFDM symbol is denoted as

$$x = \{x_{N-G,\dots,}x_{N-1,}x_0, x_{1,\dots}x_{N-1}\} \qquad (2)$$

The first G samples, as same as the last G ones of the training symbol, are added to make up of the cyclic prefix (CP).

At the receiver, the received samples are modeled as

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$$y(i) = \exp(j2\pi\varepsilon i/N+\varphi)\sum_{l=0}^{N_L-1} h_l x(i-\tau_l) + n(i) \quad (3)$$

where τ_l is the unknown arrival time of a symbol, ε is the carrier frequency offset normalized to subcarrier spacing, φ is the initial phase, n_i is the sample of zero mean complex Gaussian noise process with variance ${\sigma_i}^2$ and N_L is the number of resolvable path.

Synchronization of an OFDM signal requires finding and compensating the symbol timing and carrier frequency offset.

III. THE AVAILABLE TIMING AND FREQUENCY OFFSET ESTIMATION METHOD [7]

Samples in the preamble satisfy two conditions:

$$Condition\ 1:\ x_i = x_{i+N/2},\quad i = 0,1,\dots,N/2-1 \quad (4)$$

$$Condition\ 2:\ x_i = x_{N-i},\quad i = 1,2,\dots,N/2-1 \quad (5)$$

Such a sequence could be generated in the frequency domain by modulating only even numbered subcarriers and the modulating data are symmetric about the $N/2$-th subcarrier, i.e.,

$$X_i = X_{N-i} \quad \text{for } i = 1,2,\dots,N/2 \quad (6)$$

In the receiver, the relationship between a transmitted sample and its received version is

$$r(i) = x(i-\theta)e^{j2\pi\varepsilon i/N} + n(i) \quad (7)$$

where θ is the correct timing point.

Zhang's algorithm is divided into two modes: acquisition mode and tracking mode. In the former stage, timing and coarse frequency offsets are estimated simultaneously while in the latter stage, a fine adjustment algorithm is taken to estimate the remaining carrier frequency offset.

In the acquisition mode, define a vector based on condition 2:

$$\Psi_i = [\Psi_i(1), \Psi_i(2), \dots, \Psi_i(N/2-1)] \quad (8)$$
$$= \left[\frac{\Gamma_i(N/2-1)}{|\Gamma_i(N/2-1)|}, \frac{\Gamma_i(N/2-2)}{|\Gamma_i(N/2-2)|}, \dots, \frac{\Gamma_i(1)}{|\Gamma_i(1)|}\right]$$

where $\Gamma_i(k) = r(i+N-k)\cdot r^*(i+k)$ and:

$$\Phi_i(f) = \sum_{k=1}^{N/2-1} \Psi_i(k)\cdot e^{-j2\pi fk} \quad (9)$$

where $|f| \le 1/2$.

At the correct timing point, Equ. (8) could be simplified as follows:

$$\Psi_\theta = \left[e^{j2\pi\varepsilon\cdot 2/N}, e^{j2\pi\varepsilon\cdot 4/N}, \dots, e^{j2\pi\varepsilon\cdot(N-2)/N}\right] \quad (10)$$

From Equ. (10) it is known that $\Phi_\theta(f)$ gets its maximum value at $f=2\varepsilon/N$, i.e.,

$$\hat{f} = \underset{\mathrm{f}}{\arg\max}\{|\Phi_\theta(f)|^2\} \quad (11)$$

Accordingly, the carrier frequency offset is coarsely estimated from equation $\hat{\varepsilon} = N\hat{f}/2$.

Note that $|\Phi_i(f)|^2$ gets its maximum value when and only when both $\hat{\theta}$ and $\hat{f}$ are estimated correctly. Hence, timing and coarse frequency offset estimation could be performed at the same time by searching the maximum value in the following matrix:

$$\Lambda_{2K+1,L+1} = \begin{bmatrix} |\Phi_0(K\cdot\Delta f)|^2 & |\Phi_1(K\cdot\Delta f)|^2 & \dots & |\Phi_L(K\cdot\Delta f)|^2 \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ |\Phi_0(\Delta f)|^2 & |\Phi_1(\Delta f)|^2 & \dots & |\Phi_L(\Delta f)|^2 \\ |\Phi_0(0)|^2 & |\Phi_1(0)|^2 & \dots & |\Phi_L(0)|^2 \\ |\Phi_0(-\Delta f)|^2 & |\Phi_1(-\Delta f)|^2 & \dots & |\Phi_L(-\Delta f)|^2 \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ |\Phi_0(-K\cdot\Delta f)|^2 & |\Phi_1(-K\cdot\Delta f)|^2 & \dots & |\Phi_L(-K\cdot\Delta f)|^2 \end{bmatrix} \quad (12)$$

where $(L+1)$ is the length of the time window, Δf is the frequency resolution and $2\cdot K\cdot\Delta f$ equals the coarse estimation range of the frequency window, which is up to half the total signal bandwidth. Assume the largest element in the matrix is $|\Phi_j(i\cdot\Delta f)|^2$, then $\hat{\theta} = j$, $\hat{\varepsilon} = (N\cdot i\cdot\Delta f)/2$.

In the tracking mode, only the remaining frequency offset ε_R is left:

$$\varepsilon_R = \varepsilon - \hat{\varepsilon} \quad (13)$$

For a given N and ε, the value of ε_R is determined by that of Δf. A fine adjustment is deployed to estimate ε_R. Based on condition 1, a carrier frequency offset fine adjustment algorithm is derived as:

$$\widehat{\varepsilon_1} = \frac{\text{angle}\left\{\sum_{i=0}^{N/2-1} r(\theta+i+N/2)r^*(\theta+i)\right\}}{\pi} \quad (14)$$

which is just the Schmidl's algorithm.

Based on condition 2, ε_R could be estimated as:

$$\widehat{\varepsilon_2} = \frac{N\sum_1^{N/2-1}|D(k)|\cdot(N-2k)\cdot angle\ \{D(k)\}}{2\pi\sum_1^{N/2-1}|D(k)|\cdot(N-2k)^2} \quad (15)$$

where $D(k) = r(\theta+N-k)\cdot r^*(\theta+k)$.

Estimation range of the carrier frequency offset fine adjustment algorithm is only $\pm N/2(N-2)$ subcarrier spacing, imposing the following restriction on Δf:

$$|\Delta f| < 1/2(N-2) \quad (16)$$

When we consider both conditions 1 and 2, a fine adjustment estimator with weighted factors can be derived as:

$$\hat{\varepsilon} = \frac{\widehat{\varepsilon_1}+\rho\cdot\widehat{\varepsilon_2}}{1+\rho} \quad (17)$$

where ρ is the weighted factor and $0 \le \rho \le 1$. The larger ρ is, the more accurate $\hat{\varepsilon}$ is. When ρ is 0, the frequency offset estimator is reduced to the Schmidl's algorithm.

III. PROPOSED SYNCHRONIZATION AND FREQUENCY OFFSET ESTIMATION METHOD

Zhang's method performs the timing synchronization and coarse frequency offset estimation simultaneously. However, several factors make the algorithm unsatisfactory.

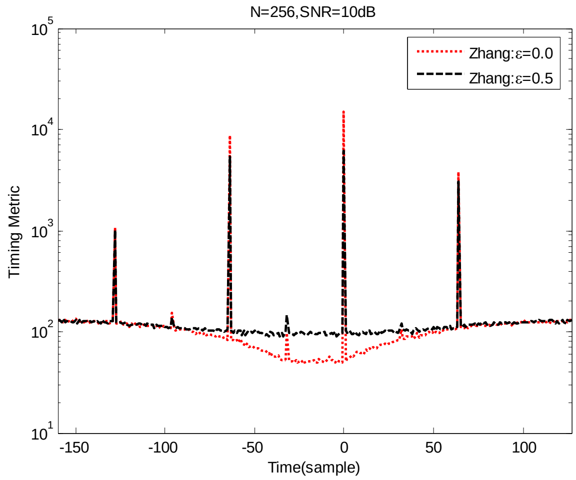


Fig. 1 Timing metric of Zhang's algorithm

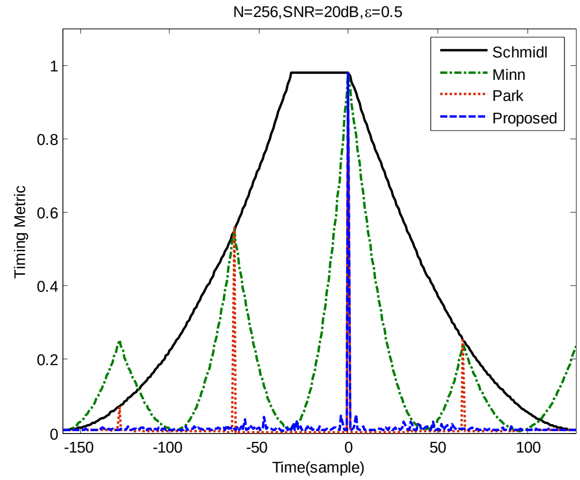


Fig. 2 Timing metric of proposed and other algorithm

(a) The timing metric has three discernible side-peaks which could not be eliminated even though ε_R is zero, as we learn from Fig. 1. These side-peaks would further result in wrong judgments.

(b) The computational load to get the matrix is too heavy to make the algorithm realizable. In all, the matrix is composed of (2K+1) (L+1) elements. Unfortunately, none of them can be calculated iteratively.

(c) The accuracy of timing estimator is greatly influenced by the remaining carrier frequency offset. From Fig. 1, when ε_R is 0, the difference between the correct timing point and side-peaks is small. The difference continually decreases as ε_R becomes larger. When ε_R is 0.5, it is quite hard to distinguish them. We know that ε_R largely depends on Δf. Thus a small Δf indicates not only a more precise timing estimator but an even heavier computational load for a fixed frequency offset estimation range.

(d) We learn from Equ. (16) that for a system with a large number of subcarrier, Δf has to be smaller, which in turn indicates a larger K and a heavier computational load. Consider a case in which N is increased from 256 to 1024 (L+1=160 and $\Delta f = 1/2(N-2)$), K has to be increased from 254 to 1022 and there would be about 300% more elements in Equ. (12).

(e) In the tracking mode, the fine adjustment algorithm works only when the correct timing point is perfectly found.

All the above factors would prevent Zhang's algorithm from working correctly and effectively. In order to improve Zhang's timing estimator performance and simplify the algorithm, a modified method based on [5] and [7] is proposed.

In the proposed method, a single training symbol is used to estimate timing and frequency offset jointly. Firstly, the starting point of the proposed preamble is determined, so that the original complex matrix would be simplified into a one dimensional one. Then the coarse frequency offset in the acquisition mode and fine adjustment algorithm in the tracking mode would be proceeded.

The Schmidl timing estimator is robust to frequency offset. Nevertheless, there is a plateau in the timing metric which results in error when we try to determine the exact timing point of the symbol. This is because the values of the timing metric around the correct starting point are almost the same [4] [5]. To enlarge the difference between two adjacent values of the timing metric, a training preamble with PN sequence weighted factor is introduced.

At the transmitter, the new preamble can be defined as

$$x_i{}' = s_i \cdot x_i,\ \ k = 0,1,\ldots,N-1 \tag{18}$$

where s_i, drawn from PN sequence with a value either +1 or -1, is the weighted factor of the i-th sample at IFFT outputs.

At the receiver, the received symbol is demodulated by the same PN sequence and the corresponding pairs of samples are correlated. The new timing metric is defined as:

$$M(d) = |P(d)|^2/|R(d)|^2 \tag{19}$$

where

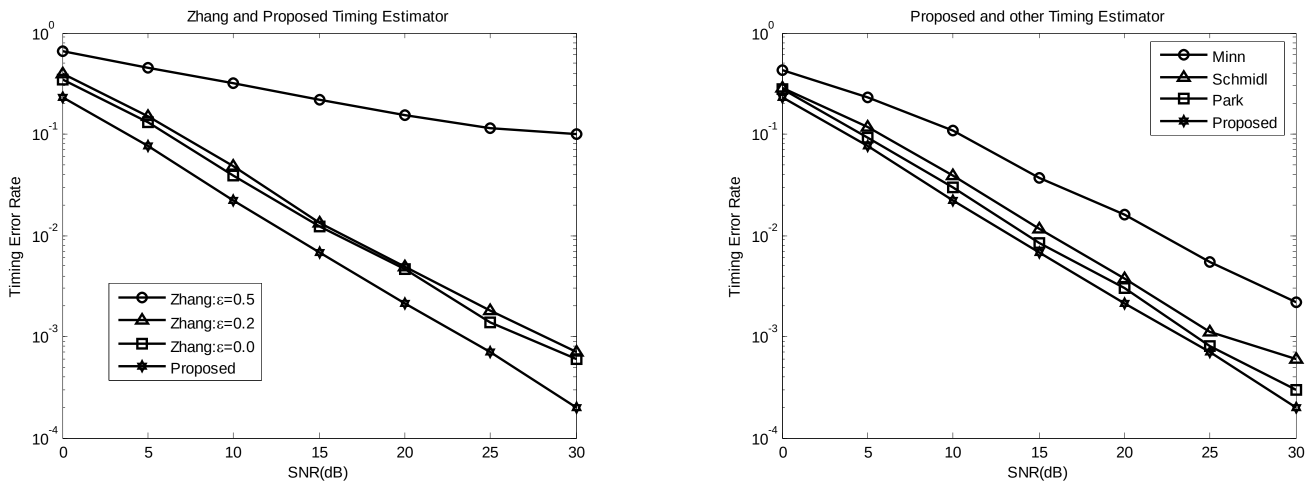
$$P(d) = \sum_{k=0}^{N/2-1} s_k s_{k+N/2} r^*(d+k)\, r(d+k+N/2) \tag{20}$$

$$R(d) = \sum_{k=0}^{N/2-1} |r(d+k+N/2)|^2 \tag{21}$$

Since the PN sequence samples are generated randomly, the influence exerted by weighted factors would be removed only at the correct timing point, which is taken as the start of the useful part of the training symbol, and values around that particular point would be relatively small.

The timing metric of proposed method in AWGN environment is shown in Fig. 2. For comparison, Schmidl, Minn and Park timing estimators are also presented. The frequency offset is set to be 0.5 subcarrier spacing but it does not exert much influence on the timing estimators as they are robust to it. The correct timing point is indexed 0 in the figure. As expected, the proposed timing metric shaped like an impulse has neither a plateau nor side-peaks, making a more accurate timing synchronization possible.

As soon as the correct timing point, i.e., θ, is located, Equ. (9) could be calculated to coarsely estimate the frequency offset. Stated in another way, to get $\hat{f}$, we now only need to calculate one column of Equ. (12) and look for the maximum value.



(a) (b)

Fig. 3 TER comparison of different timing estimators

Then in the tracking mode, the received samples are compensated with coarse frequency offset, and the fine adjustment algorithm (Equ. (17)) is taken to estimate the remaining frequency offset ε_R. The modification to the original timing estimator does not in the least influence the coarse and fine frequency offset estimation.

Overall, the proposed algorithm has some attractive features superior to Zhang's method.

(a) The computational complexity is significantly reduced because the timing synchronization is performed in the beginning and the original complex matrix is simplified into one dimension. That is, only one column containing ($2K$+1) elements is needed in the acquisition mode, far smaller than that of Zhang's timing estimator, which is ($2K$+1) (L+1).

(b) The proposed timing estimator is robust to frequency offset, so it would not be affected by the value of Δf. To further simplify the algorithm, K could be assigned a smaller value by increasing Δf slightly as long as the fine adjustment algorithm works. For instance, if Δf is increased from 0.0025 to 0.003875, K would be 53.9% smaller, i.e., it is decreased from 200 to 130.

(c) The PN sequence weighted factors lead to an impulse shaped timing metric without plateau and side-peaks, making an accurate timing estimator with less error variance possible.

(d) The proposed algorithm exerts no influence on the coarse and fine frequency offset estimation.

Consequently, the proposed algorithm has the virtue of being precise and affordable. The timing performance and computational complexity is further illustrated by computer simulations.

IV. SIMULATIONS AND ANALYSIS

In this paper, a wireless system operating at 5 GHz with bandwidth of 5 MHz and *Maximum Doppler Shift* of 48.1 Hz is assumed for simulations. An outdoor dispersive, fading channel is modeled as 5 independent Rayleigh-fading paths with path delay τ_i of 0,3,5,9, and 12 samples and path gains given by h_i=exp($-\tau_i$). The length of the preamble is 256 and that of CP is 32. QPSK modulation is employed and 10000 simulations are run.

A. *Timing Synchronization Analysis*

As we know, perfect timing point estimation is crucial for the operation of the fine adjustment algorithm. In Fig. 3, the performance of different timing estimators are evaluated in terms of timing error rate (TER). For Schmidl [1] and Minn [3] algorithm, TER is equal to the probability that the estimated starting point is outside CP. For Park [4], Zhang [7] and the proposed algorithm, TER is the probability that the estimated starting point is not the correct timing point.

Fig. 3(a) indicates that Zhang's timing estimator is very sensitive to remaining frequency offset and has a large TER even though the frequency offset is perfectly estimated (ε_R=0) in the acquisition stage. By comparison, the proposed algorithm is more accurate than Zhang's timing algorithm. When SNR is low, improvement about 3dB and 4dB of the proposed timing estimator compared to [7] could be obtained when the remaining carrier frequency offset ε_R equals 0.0, 0.2, respectively. With SNR of 15dB, TER of proposed algorithm is 0.0067, while TER of Zhang's method is 0.0122, 0.0134 and 0.2177 when ρ equals 0.0, 0.2 and 0.5, respectively. This is due to the improved timing estimator robust to ε_R and free from side-peaks.

From Fig. 3(b) it is also clear that the proposed algorithm has better performance than Schmidl's, Minn's and Park's. When SNR is low, improvement about 1dB and 2dB of the proposed timing estimator compared to [4] and [1], respectively, could be obtained. When SNR is 15dB, TER of proposed, Schmmidl, Minn and Park algorithm is 0.0067, 0.0116 0.0371 and 0.0085.This is because:

(a) It does not have a plateau created in [1].

(b) It does not have side-peaks inherent in [3] and [4].

(c) The timing metric is shaped as an impulse, which is better than [3] even though side-peaks are not taken into account.

Then in the tracking mode, the fine adjustment algorithm is adopted to get an estimation of ε_R more accurate than that of Schmidl. In the AWGN channel, performance improvement about 1 dB of the fine adjustment algorithm compared to Schmidl's algorithm is obtained when ρ is 1, while in multipath fading channel, the improvement would be up to 4.6dB in a low SNR [7].The improved timing synchronization algorithm makes Zhang's frequency offset estimation method feasible and reliable.

B. Computational Complexity Analysis

Detailed analysis on computational complexity listed in Table 1 illustrates the proposed algorithm's efficiency. The computational complexity of Zhang algorithm is significantly reduced by preceding the timing synchronization since it enables the original matrix to be simplified into one dimension. On one hand, it saves considerable time to perform the complex algorithm. On another hand, it reduces the overall system cost by decreasing the number of devices such as expensive memory elements. For example, when N=256, K=160, L+1=160, Δf=0.003125, the calculation amount of complex multiplication is 7866.3% less than that of the original algorithm. Note that in the table, comp mul, comp div, comp add, exp oper and mode oper means complex multiplication, complex division, complex add, exponential operation and mode operation, respectively.

Table 1. Computational Complexity Analysis

	Zhang	Proposed
comp mul	(2K+2)(N/2-1)(L+1)	(K+1)(N-2)+(L+1)(N+1.75)
comp div	(N/2-1)(L+1)	(N/2+L)
comp add	(2K+1)(N/2-2)(L+1)	(2K+1)(N/2-2)+(L+1)(N-2)
exp oper	(2K+1)(N/2-1)(L+1)	(2K+1)(N/2-1)
mod oper	(N/2+2K)(L+1)	(N/2+2K)

V. Conclusions

An efficient joint frame timing and frequency offset estimation algorithm for OFDM systems as a modification to Zhang's method has been proposed. By using PN sequence weighted factors, an impulse shaped timing metric robust to frequency offset and free from side-peaks is obtained. Performing timing synchronization first, we are able to simplify Zhang's algorithm significantly with better timing synchronization performance and no loss in the accuracy of frequency offset estimation. Simulation results have proved the accuracy and efficiency of the proposed algorithm.

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